

MODAL INVESTIGATION OF A DAM

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ABSTRACT. Procedures and results of an experimental and analytical Modal investigation performed in 1996 on a cylindrically shaped 170 m long and 46 m high reinforced concrete dam in Sweden are presented.

A preliminary Ambient Vibration Test was performed on the dam in April to approximately determine its fundamental natural frequency. This was necessary to optimize the equipment and procedures to be used for the main tests.

The main investigation consisted firstly of a Forced Vibration Modal Test performed on the dam in June. Excitation was provided by a servo-hydraulic vibration generator. This induced random-type horizontal forces in the upstream-downstream direction at the dam crest. In a first phase, the dynamic response (acceleration) was measured in three directions in a total of 227 points located on the dam crest and downstream face. The measurement point grid was then extended to parts adjacent to the dam: rock, spillway and inlet/powerhouse structures. This increased the number of measurement points to 270. Processing of the measured signals yielded twelve natural frequencies in the range $f = 3.55 \dots 12.91$ Hz and the corresponding mode shapes and damping. All modes exhibit comparatively low damping coefficients in the range $\zeta = 1.05 \dots 1.74\%$ of critical.

Secondly, a Finite Element model of the system (dam plus adjacent structures) was created and updated on the basis of the test results. Model updating yielded good correlation for eleven of the twelve modes identified although no attempts were undertaken to include neither dam-reservoir nor dam-soil interaction effects in the theoretical model. The MAC-value for one of the mode pairs was $MAC = 59\%$ only. This is due to cross-talk between the first vertical bending mode and a closely spaced horizontal bending mode. As no information on damping was included in the Finite Element model effects of this kind could not be covered.

It could be concluded from the results of the investigation that the boundary conditions at most of the dam/rock connections are elastically clamped-in. As an exception, practically stiff clamping-in is valid for the vertical dam/rock connection at the dam north abutment. Some dynamic movement was observed at the dam south abutment as well as at the spillway piers and the inlet/powerhouse structures. These movements were however small compared to the movement of the dam crest.

The goal of the investigation could hence be reached: An updated FE model is now available for further analytical calculations.

1. INTRODUCTION

Commissioned by and in co-operation with Vattenfall Utveckling AB, EMPA performed an experimental and analytical System Identification on a dam in Sweden in 1996. Due to the rather high costs of these tests they were also subject of an EMPA Research Project and hence also financed by EMPA.

The dam will be referred to as "Norsjö Dam" throughout this paper. After a preliminary Ambient Vibration Test performed in April, the dam was subject of a Forced Vibration Modal Test in the second half of June. Subsequently, a Finite Element model of the dam including the adjacent structures was created. As a last step in the procedure, this Finite Element model was updated based on the Forced Vibration Test results.

2. THE DAM

The Norsjö Dam is a cylindrically shaped reinforced concrete structure with a length at the crest of 169.4 m and a maximum height of roughly 46 m (Figs. 1 and 2). The radius of curvature of the dam upstream face is $R = 110$ m. The dam width is 2.5 m at the crest and the top ~7 meters of the dam and then increases continuously to 5.5 m at the bottom of the structure. The dam upstream face being vertical this results in a slightly tilted downstream dam face. Construction of the dam was performed by pouring the concrete in vertical strips of 10.6 m width and leaving open intermediate 1.4 m wide gaps. These gaps were filled with concrete after the first, important part of shrinkage had been completed in the 10.6 m-strips. Although, due to the reinforcement, the dam can be considered as a monolithic structure it can also be considered as consisting of 14 blocks of 10.6 m plus 1.4 m width each. This block structure was used when defining the measurement point grid.

On the dam downstream side a reinforced concrete wall of 150 mm width is located at a 0.8 m distance of the main structure. This wall and the dam are connected through a large number of reinforced concrete bars of 150 x 150 mm cross section positioned in a grid with a mesh width of 2.88 m. Wooden planks are placed on every second row of these concrete bars which makes the open space between the dam and concrete wall and hence the whole dam downstream face easily accessible. The vertical connection between the plank-footpaths is provided through ladders.

At the dam crest, the dam main structure and the secondary concrete wall are connected through a 4 m wide horizontal reinforced

concrete slab of roughly 300 mm thickness. These elements are connected through reinforcement bars.

The dam is founded in rock along its north side vertical boundary and along its bottomline. The south side vertical dam boundary is connected to the first of three large triangular shaped reinforced concrete spillway piers with the two spillways in-between the piers. This first pier has a width of 6 m whereas the middle and the outer pier are 3.2 m and 2.6 m wide respectively. Adjacent to this spillway construction are two inlet/powerhouse structures. The south side system boundary is an earthfill dam.

The Norsjö Dam is part of a system of several dams and reservoirs located on a river over its whole length. The power production regime is typical for a river exploitation system where the powerhouses are located close to the dams' foundations and the difference in height between reservoir level and turbines is comparatively small. The reservoir level is quite stationary at all times. These facts were to be kept in mind when performing System Identification Tests on the dam.

3. PRELIMINARY AMBIENT VIBRATION TESTS

Considering the distance between EMPA and the dam site and the hence limited possibilities to change main test parameters as e.g. the means of excitation during the tests, preliminary Ambient Vibration Tests were performed. The goal of the tests was to get an idea of the dam fundamental natural frequency. This was of some concern because EMPA disposes of two excitation systems with different capabilities in low-frequency excitation.

The structural vibrations excited by ambient sources as wind, waves and microseismicity were measured using three Kinematics accelerometers and analysed in the frequency domain. Data processing was performed on-site using an analog-to-digital converter, a laptop computer and specific software.

In a total of seven measurement set-ups, the ambient dam vibrations were measured at three locations on the dam crest, in the region of the spillways, the inlets and the adjacent earthfill dam. The net testing time was several hours. The main results of the preliminary tests were:

- The dam fundamental natural frequency could be expected to be $f \approx 3.2$ Hz,
- Operation of the two turbine/generator units resulted in significant peaks in the frequency spectra measured.

Considering the low frequency of the dam fundamental mode it was decided to use the more powerful of the two EMPA vibration generating devices for the main tests.

Considering the disturbing effects of the turbine/generator units when operating it was decided to perform the main tests during the times when these were shut down. This meant that a certain cooperation with the power station control center would be necessary and that measurements could be performed during weekends and nights only.

4. MAIN FORCED VIBRATION MODAL TESTS

4.1 The Operation

The roughly 12 t of EMPA hydraulic and electronic equipment was transferred to the dam site with a tractor-trailer. The EMPA personnel, a total of nine people, flew and drove to the site June

19/20, 1996. Installation of the test equipment took place June 21. The aluminum shelter containing the electronics was deposited at the dam north end and the shaker and hydraulic power pack at the pre-determined positions (refer Paragraph 4.2) on the dam crest. These operations were performed with the tractor's on-board crane and with the help of local power station staff operating a huge fork-lift. Modal Tests were performed June 22 and 23 and during the nights of June 24 to 27. Further tests which are not subject of this paper were performed June 28. The equipment was loaded back to the tractor-trailer June 29 and the EMPA team left the dam site June 30.

The operation could be performed without any accident or severe problem to be surmounted.

4.2 The Excitation

A servohydraulic vibration generator was used to excite the dam in the horizontal upstream-downstream direction. The main element was a servohydraulic cylinder with a maximum force amplitude $F = \pm 32$ kN. The cylinder stroke is 250 mm and it is equipped with a 63 l/min two-stage servo-valve. Steel plates were mounted on top of the piston rod, resulting in a horizontally moving mass of 1'000 kg (Fig. 3).

The cylinder is fixed to a supporting steel frame. A load cell is located between the cylinder footing plate and the supporting frame. A rigid connection between the supporting frame and the structure to be excited is provided by ten M20 screws and the corresponding dowels. Fixing of the shaker to the dam crest structural concrete proved to be quite difficult but could be completed in one day.

A hydraulic power pack supplied 80 l/min of oil with a 280 bar pressure. An air-cooled heat exchanger was connected to the hydraulic power pack oil tank to keep the oil temperature in the 50°C to 60°C range. The length of the flexible pipes transporting the pressurized oil from the power pack to the shaker being not longer than 20 m, the power pack could not be located outside the dam. It was therefore placed on air-springs. Experience had shown that without these soft springs high-frequency vibrations generated by the power pack would severely disturb the shaker induced structural vibrations. Due to the same reason the oil pipes between the hydraulic power pack and shaker were isolated from the dam structure through foam rubber layers.

The power consumption of the whole shaker system is about 90 kW. For field tests in Switzerland, EMPA uses a mobile Diesel aggregate to drive the system. However, for the tests on the Norsjö Dam, the power was provided by the local power station.

The cylinder was driven displacement-controlled with a Schenck Series 31 control system. The cylinder maximum force amplitude, $F = \pm 32$ kN, is reached for frequencies above $f = 2.0$ Hz. The control signal used for the tests on the Norsjö Dam was of a specially designed band-limited random-type. The force spectrum was tuned to be flat in the desired frequency range $f = 3.0 \dots 16.0$ Hz. The basic shape of the excitation spectrum was set by optimisation of the (PID) parameters of the cylinder control loop. The excitation spectrum fine tuning was achieved by electrical filtering of the control signal so that the force spectrum finally met the requirements (Fig. 4). These tuning operations were performed during the pre-test preparation of the equipment at the EMPA laboratories.

Another problem to be solved during the pre-test phase was the choice of the optimum driving point. This choice was based on the natural frequency and mode shape calculations performed by a team of the Stockholm Royal Institute of Technology using a Finite Element model [1]. This model was based on the known dam geometry and on the assumption that the connection between dam and adjacent rock and structures would be somewhere between simply supported and completely clamped-in.

Point # 6 of the measurement point grid (refer next Paragraph) was chosen as a probably good driving point. After having concluded the tests for all measurement points on the dam crest during the first day of testing a preliminary data processing and analysis were performed. The results of these tests were then compared with the abovementioned Finite Element Analysis results. As a consequence, the driving point was rated as being optimal. This was very fortunate because moving of the shaker to a different location would have meant another day of work.

4.3 Response Measurements

The dam vibrations induced by the shaker operation were measured as accelerations. In three cases, a measurement unit consisted of three accelerometers Brüel&Kjær 8306 (sensitivity: 10 V/g, resolution: 10^{-6} m/s²) mounted orthogonally to each other on a supporting steel plate being rigidly fixed to the dam structure with the help of screws (Fig. 5). The fourth measurement unit consisted of three accelerometers PCB 393B31 (specs similar to those of the B&K 8306). Hence, a total of twelve response signals plus the force signal were simultaneously acquired in one measurement cycle.

In the pre-test phase the dam was considered to be a monolithic structure and a uniform measurement point grid was hence prepared in the data acquisition and processing computer. This assumption had however to be validated on-site. Correspondingly, a first test with placing the accelerometer units on the two sides of a joint between a block and intermediate strip (refer Chapter 2) was performed. This yielded that no relative movement was to be observed in these joints. This was no surprise considering the fact that the dam is monolithically reinforced all over the structure. In Switzerland, relative movements in joints of a non-reinforced concrete gravity dam have been observed [2]. Subsequent to this first test, it was decided to treat the dam as a monolithic structure and to stay with the prepared measurement point grid.

The dam response was measured at 227 points distributed over the crest and the dam downstream face at the inspection galleries in three directions (Fig. 6). The grid point positions on the dam body were uniformly distributed by selecting two measurement points per block at 4 m distances from the block centerline on the dam crest and repeating this pattern for all the inspection galleries. After completion of this measurement point grid, the grid was extended to the rock foundations and to the spillway and inlet/powerhouse structures where accessible. The number of response measurement points thus increased to 270. Figure 7 gives the measurement points added at the spillway and inlet/powerhouse structures.

4.4 Signal Acquisition and -processing

An EMPA-developed amplifier provided the signal necessary to drive the accelerometers and amplified the incoming signals. Subsequent signal conditioning (anti-alias filtering, after-filtering amplification), 16-bit-digitization and signal acquisition was

performed with the help of a DIFA/SCADAS front-end controlled by a HP 725/100 computer and using CADA-X.

The technical details of signal acquisition were:

- Sampling rate $s = 100$ Hz,
- Block size: 4096 samples, equalling a time window length of roughly $T = 41$ s,
- Windowing: Hanning on all time windows (response and excitation),
- Frequency spectra range $f = 0...25$ Hz,
- Frequency resolution $\Delta f = 0.024$ Hz,
- Frequency Response Functions calculated from the averaged spectra derived from six consecutive time windows.

The continuous band-limited random-type force control signal covering 100% of the time window was generated using the DIFA/SCADAS front-end. The signals from four three-dimensional measurement points plus the driving point force signal were acquired simultaneously, Fourier transformed and recalculated to obtain a leakage-free estimate of the frequency response functions.

The driving point Frequency Response Function, FRF, as shown in Figure 8 indicates twelve natural frequencies of the dam in the range $f = 3.0...13.5$ Hz.

4.5 Results of Modal Parameter Estimation

No disturbing effects due to instationary environmental conditions leading to system non-linearities were to be observed. Water level variations in the reservoir were less than 10 mm during a given testing period and less than 100 mm for the whole week of testing. The reservoir water temperature was nearly constant at 12°C. Air temperature was in the 11...15°C and 7...11°C ranges at daytimes and during the nights respectively.

Modal parameters of the Norsjö Dam were estimated in two steps using CADA-X. Pole values (damped natural frequencies, damping ratios) and modal participation factors were calculated using the Least Square Complex Exponential Algorithm for the single input case. A helpful means for the evaluation of the number of physical modes is the Modal Indicator Function calculated from all measured FRF's (Frequency Response Functions) as given in Figure 9. This function shows minima at the natural frequencies, implying that the in-phase response energy of the dam is a minimum when it is vibrating at resonance. The twelve modes of the dam, as indicated by the Modal Indicator Function significantly contribute to the dynamic response of the dam in the selected frequency range $f = 3.0...13.5$ Hz. This was confirmed by an exact analysis based on non-linear weighted residues of the driving point FRF in this frequency range.

The natural frequencies and damping ratios were extracted from the measured FRF's. The accuracy of the estimated natural frequencies is within 2% and the accuracy of the estimated critical damping ratios within 5%. The quality of the estimated modal parameters was investigated by correlating the measured and synthesized FRF's on the basis of estimated modal parameters. The correlation coefficient between these functions exceeds 97%. The values of the natural frequencies and corresponding damping coefficients given in percent of critical as well as a discussion of the mode shape characteristics are presented in Table 1.

The contents of the column "Mode Characteristics" given in this Table is based on the mode shapes. Due to space restrictions the crest shapes can be shown in Figure 10 only. As a consequence of the structure's complexity detailed mode shape identification had to be based on the shape's animated display on the computer screen.

These mode shapes are firstly rated on the horizontal shape as valid for the dam crest (Fig. 10). This especially applies to the "number of nodes" given and to the expression "symmetric" and "antimetric" horizontal bending respectively. Secondly, the stability of the shape over the dam height is discussed. "Stable" means that no phase changes can be determined when checking the mode shape in the vertical direction (\Rightarrow horizontal bending). A vertical bending mode is identified in the case of a clear change in phase. In addition, there are cases, where basically horizontal bending modes show signs of overlaid vertical bending (shape "unstable" over the dam height).

Mode Nr.	Freq. f [Hz]	Damping ζ [%]	Mode Characteristics
1	3.55	1.08	AHB1 , one node, stable; strong
2	3.64	1.63	SHB1 , two nodes, stable; weak
3	4.44	1.13	as SHB1 , two nodes, stable; but vertically more bent towards the air side than for mode 2; strong
4	5.00	1.26	AHB2 , three nodes, stable; moderate
5	6.45	1.24	SHB2 , four nodes, stable; moderate
6	7.89	1.61	VB1 , no nodes, phase change; weak
7	8.31	1.33	AHB3 , five nodes, unstable; influences of VB1? (dam crest stronger bent towards the water side than could be anticipated); moderate
8	8.85	1.29	VB2 , one node, phase change; moderate
9	9.97	1.74	VB3 , two nodes, phase change; weak
10	10.51	1.20	SHB3 , six nodes, stable; strong
11	11.48	1.40	VB4 , three nodes; unstable; none
12	12.91	1.05	AHB4 , seven nodes, stable; strong

Table 1 Norsjö Dam natural frequencies, damping and mode shapes: **A** = antimetric, **S** = symmetric; **HB** = horizontal bending, **VB** = vertical bending. The last line in the column "Mode Characteristics" indicates the intensity of the spillway pier's movement.

For all modes, the crest shape indicates the dam being simply-supported at the spillway side and completely clamped-in at the north side resulting in a slight shift of the "axis of symmetry" to the spillway or south side of the dam. This is not especially mentioned in the column "Mode Characteristics". The vertical shape of the modes indicates the boundary condition rock/structure at the dam bottomline to be elastically with being

closer to simply-supported than to fully clamped-in. Concerning participation of the spillway piers, this is in general stronger for horizontal bending than for vertical bending modes.

5. FINITE MODELING AND UPDATING

5.1 Preliminary Model

The structural system "dam plus the adjacent spillway and inlet/powerhouses structures" was modeled using the finite element program MARC. The model consisted of 3'192 elements:

- 1'041 solid elements (Type 21) with 20 nodes each representing the dam main structure and the spillway and inlet/powerhouses structures,
- 1'458 plate elements (Type 75) representing mainly the secondary concrete wall on the dam downstream side and
- 693 beam elements (Type 98) mainly for the connections "main structure-secondary concrete wall".

The geometry of the model was taken from the construction drawings provided by Vattenfall Utveckling AB. The Young's modulus was selected as $E_C = 2.4 \cdot 10^{10}$ N/m² for concrete and $E_S = 2.1 \cdot 10^{11}$ N/m² for steel. The Poisson's ratios were chosen to $\nu_C = 0.2$ and $\nu_S = 0.3$, the material density to $\rho_C = 2'500$ kg/m³ and $\rho_S = 7'850$ kg/m³ for concrete and steel respectively.

The boundary conditions in the structure/rock contact areas were modelled using three elastic springs with a stiffness of $k = 10^{11}$ N/m in the x-, y- and z-direction in each of the respective model nodes, i.e. the eight nodes provided by the MARC Element 21 plane of concern. This spring stiffness was selected after some trials during the optimization process (refer next paragraph).

5.2 Model Updating

For correlation purposes the program FEMTools was applied on the preliminary Finite Element Model making use of the forced vibration modal test results. With minor changes in the modulus of elasticity and stiffness of the springs modeling the soil-structure connection it was possible to reach a quite good coincidence between measured and calculated results. (Dam/reser-voir interaction was not considered). The boundary conditions have been found to be less sensitive than expected. To quantify and visualize this correlation between the respective mode shapes the MAC matrix was evaluated (Tables 2 and 3).

Mode pair	FEA Nr.	Freq. [Hz]	EMA Nr.	Freq. [Hz]	MAC [%]
1	1	3.66	1	3.55	87.4
2	2	3.71	2	3.64	79.5
3	3	4.64	3	4.44	95.8
4	4	4.92	4	5.00	94.7
5	5	6.16	5	6.45	89.2
6	6	7.70	7	8.31	58.6
7	9	8.13	6	7.89	83.1
8	10	8.85	8	8.85	86.1
9	11	9.48	10	10.51	72.8
10	12	9.75	9	9.97	77.2
11	13	10.81	11	11.48	74.6
12	14	11.58	12	12.91	69.4

Table 2 Comparison of the shapes of the lowest 12 dam modes as determined analytically (FEA) and experimentally (EMA).

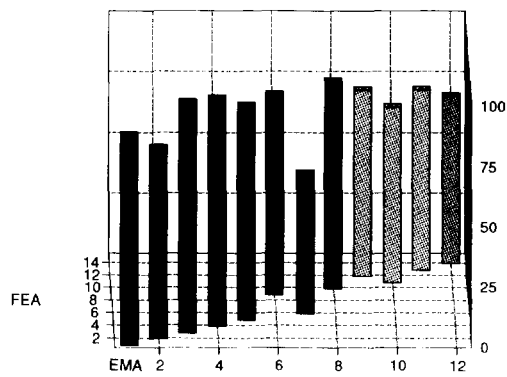


Table 3 Graphic representation of the MAC-Matrix (Table 2)

As can be seen from Table 2, the MAC correlation of the 12 modes obtained from the experimental modal analysis and their counterparts computed by the FE analysis ranges from 59% to 96%. As an example, Figure 11 shows the results for Mode pair 9. Prior to any further updating steps for the Norsjö dam model this should be validated through comparison of the experimental and analytical results for one of the forced vibration tests performed.

6. DISCUSSION

The means chosen for performing a Forced Vibration Test on the Norsjö Dam were surely as optimal as possible. Twelve natural modes of this quite complex structural system could be identified. Some borders dictated by basic physics could however not be crossed. Whereas some of the natural modes could be identified without any doubt as being horizontal or vertical bending modes this did not apply for some closely spaced modes. Here, influences of neighbouring modes with quite different shapes had to be noted. Problems of this kind do not appear if a structure's natural frequencies are well separated from each other. In the case of the Norsjö Dam, as a consequence of similar flexural stiffness in horizontal and vertical direction, the basic problem arises that

closely spaced modes of vertical and horizontal bending do exist. The most important example is:

- first vertical bending mode $f_6 \approx 7.9$ Hz,
- third antisymmetric horizontal bending mode at $f_7 \approx 8.3$ Hz

For a damped system, interaction between the corresponding mode shapes occurs when performing a forced vibration test. A Finite Element model, however, does not account for such interaction unless damping is included (which was not the case here). This is the basic reason for the relatively low MAC-value for pair 6, $MAC = 59\%$, where the FE model yields a simple horizontal bending mode, $f_6 \approx 7.7$ Hz, whereas the experimental shape of $f_7 \approx 8.3$ Hz is definitely influenced by the neighbouring vertical bending mode $f_6 \approx 7.9$ Hz. The model updating procedure was not able to cope with this problem under the circumstances given.

Considering the high MAC-values obtained for the other mode pairs, especially for the first five pairs of horizontal bending modes, where the sequence of the experimental and analytical modes is identical and the modal frequencies are quite close to each other, the Finite Element Model finally determined can be taken as optimal. It should be appropriate enough to be used for further studies of the dam behavior under any kind of load.

7. REFERENCES

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- [2] Cantieni, R., Deger, Y., Pietrzko, S., Modal Analysis of a Concrete Gravity Dam: Experiment, Finite Element Analysis and Link. Proc. 12th International Modal Analysis Conference (IMAC), Honolulu, Hawaii, pp. 442-448 (1994).

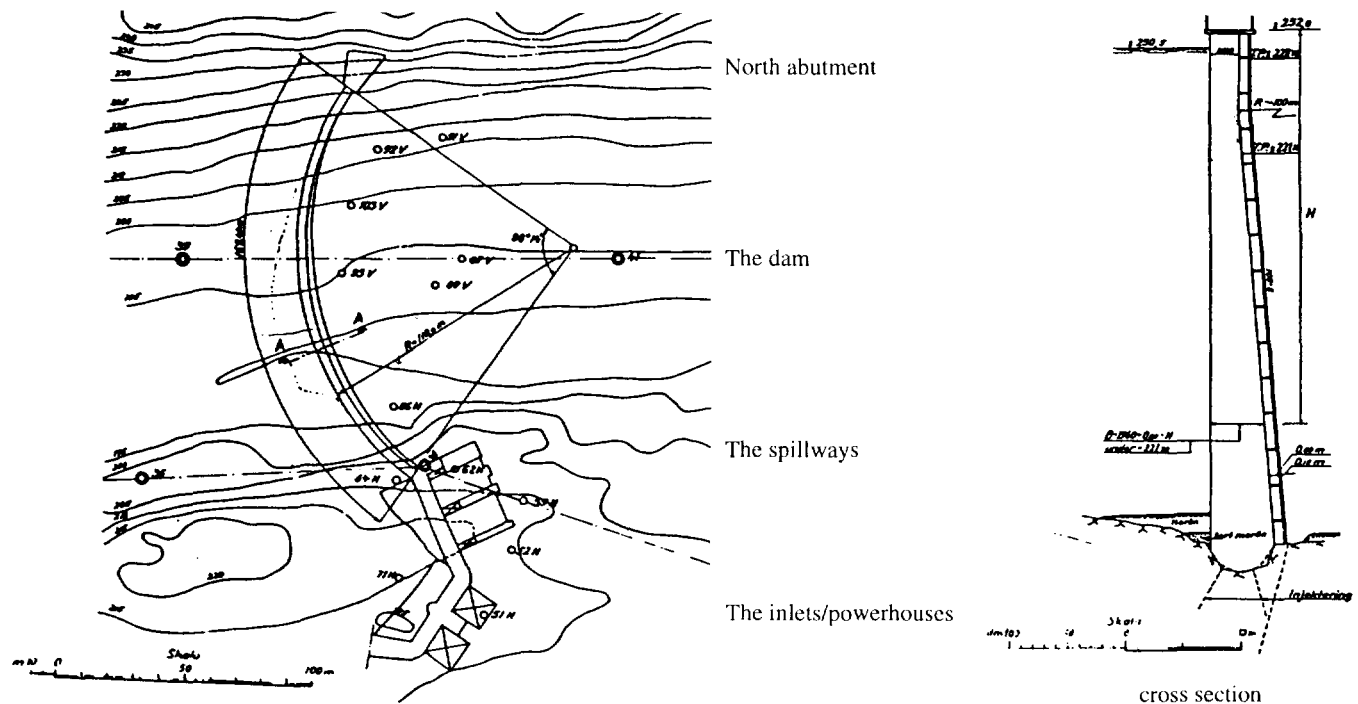


Fig. 1 Plan view of the dam and the adjacent structures; dam cross section.

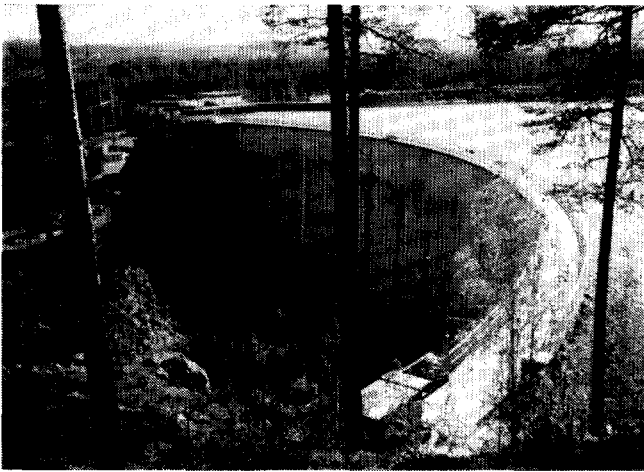


Fig. 2 Photograph of the dam as seen from North to South.

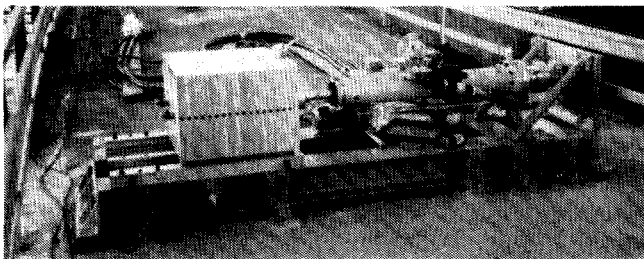


Fig. 3 The shaker is fixed to the dam.

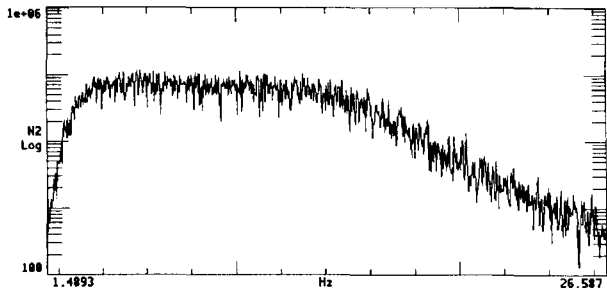


Fig. 4 Autopower spectrum of the excitation force.

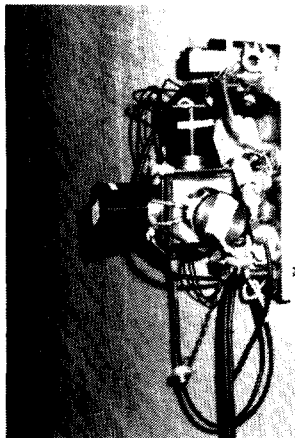


Fig. 5 Three-axial B&K 8306 acceleration measurement device mounted on the dam downstream face.

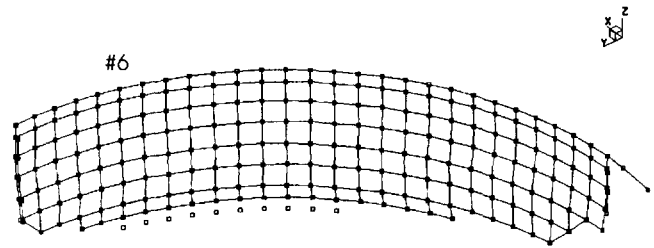


Fig. 6 Measurement point grid of the Norsjö Dam as seen from the downstream side. The shaker (single input) was located at point #6.

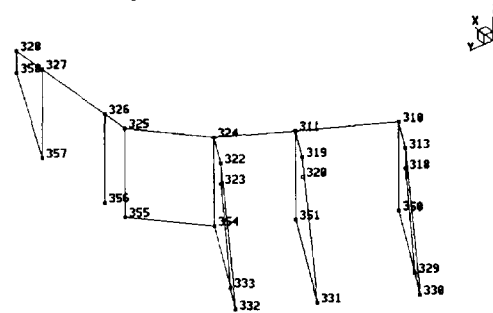


Fig. 7 The measurement point grid selected on the spillway piers and inlet/powerhouse structures (seen from the downstream side, to be added to the lefthand side of the grid shown in Figure 5).

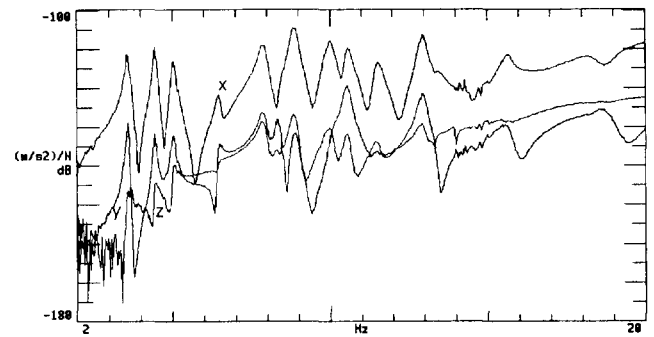


Fig. 8 Driving point #6 FRF (for x-, y-, and z-directions) indicating the existence of 12 modes in the frequency range $f = 3.0 \dots 13.5$ Hz.

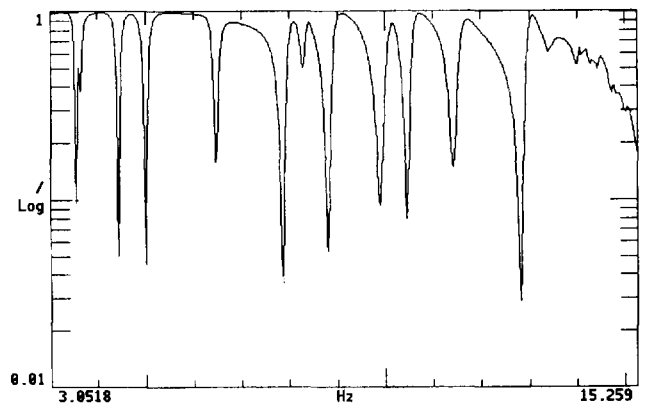


Fig. 9 Modal Indicator Function for the Norsjö Dam.

Mode 1 AHB1 3.55 Hz 1.08%	Mode 2 SHB1 3.64 Hz 1.63%	Mode 3 as SHB1 4.44 Hz 1.13%
Mode 4 AHB2 5.00 Hz 1.26%	Mode 5 SHB2 6.45 Hz 1.24%	Mode 6 VB1 7.89 Hz 1.61%
Mode 7 AHB3 8.31 Hz 1.33%	Mode 8 VB2 8.85 Hz 1.29%	Mode 9 VB3 9.97 Hz 1.74%
Mode 10 SHB3 10.51 Hz 1.20%	Mode 11 VB4 11.48 Hz 1.4%	Mode 12 AHB4 12.91 Hz 1.05%

Fig. 10 Modes shapes at the crest as determined experimentally (North dam abutment to the right; Shape type abbrev. ref. Table 1).

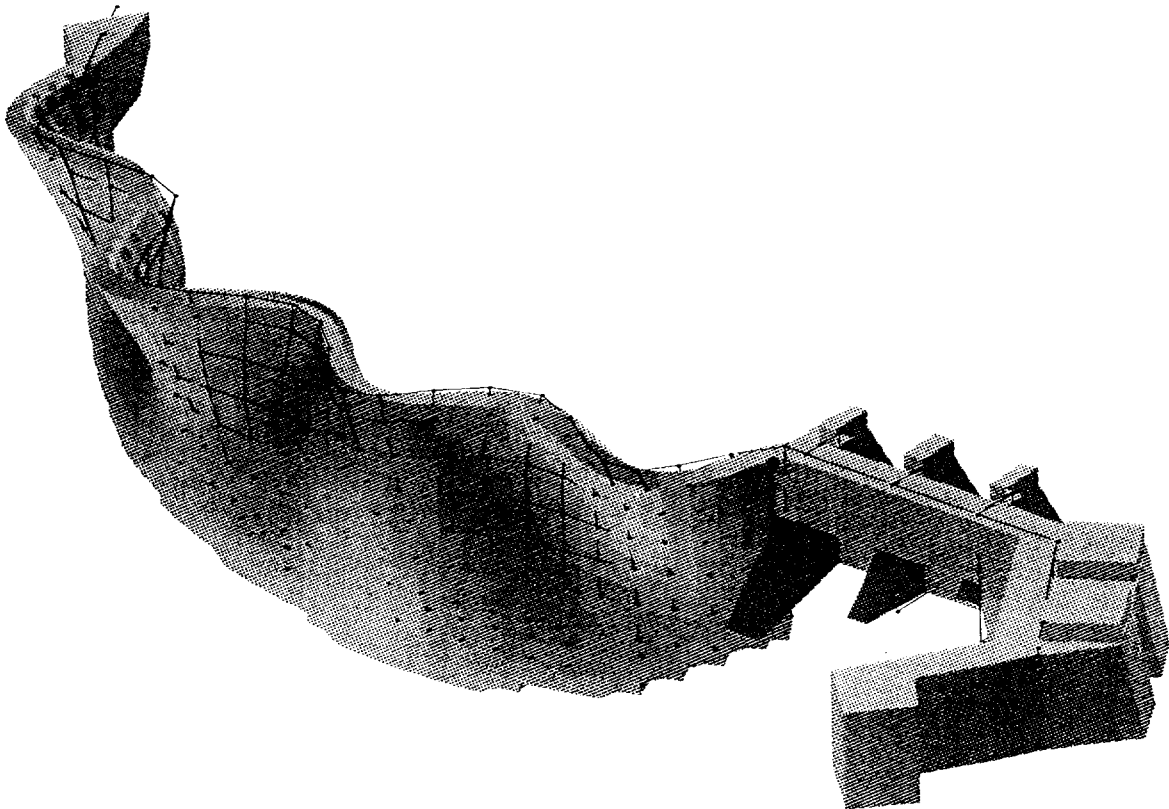


Fig. 11 Comparison Experiment/FE calculation, North dam abutment to the left, shape with 2π phase lag vs. Fig. 10: Mode pair 9, FEA Nr. 11, $f = 9.4$ Hz (shape: grey shaded), EMA Nr. 10, $f = 10.5$ Hz (shape: black dots and solid lines), MAC = 74%.